The Christofel determinantal equation for the velocities of propagation along the x axis of bismuth is

$$\begin{vmatrix} d_{11}-\rho v^2 & 0 & 0 \\ 0 & d_{66}-\rho v^2 & d_{15} \\ 0 & d_{15} & d_{55}-\rho v^2 \end{vmatrix} = 0, \tag{17}$$

and that for propagation along the y axis of bismuth is

$$\begin{vmatrix} d_{66}-\rho v^2 & 0 & 0 \\ 0 & d_{11}-\rho v^2 & -d_{15} \\ 0 & -d_{15} & d_{55}-\rho v^2 \end{vmatrix} = 0. \tag{18}$$

These particular cases are quite simple and differ from the corresponding equations for the Voigt theory in that d_{55} replaces c_{44} and d_{15} replaces c_{14}.

From Eq. (17), we have

$$d_{55}+d_{66}=\rho v_2^2+\rho v_3^2 \tag{19}$$

and, from Eq. (18)

$$d_{11}+d_{55}=\rho v_4^2+\rho v_6^2. \tag{20}$$

Furthermore, $d_{11}=c_{11}$ and $d_{66}=c_{66}$.

The sum, $v_2^2+v_3^2-v_5^2$, is $1.145\pm 0.034\times 10^{10}$ cm^2/sec^2, which is to be compared with the directly observed v_8^2, $1.153\pm 0.017\times 10^{10}$ cm^2/sec^2. It is clear that d_{44} and d_{55} cannot differ by more than 4%. Equation (20) does not yield as small an estimate for the possible difference between d_{44} and d_{55} owing to the large error introduced by the uncertainty in c_{11}.

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Dislocations in Indented Magnesium Oxide Crystals

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Dislocation rosette patterns produced by spherical and pyramidal indentors on the cleaved surfaces of magnesium oxide crystals were studied in detail. The three-dimensional arrangement of dislocation loops as deduced from the two-dimensional etching patterns is discussed. Cracks formed on $\{110\}_{90}$ planes around pyramidal indentations are believed to be due to the interaction of dislocations on $\{110\}_{45}$ planes. The temperature dependence of hardness was found to be related to the widening of dislocation bands, rather than to the distance of travel of leading dislocations. Some observations were also made on the pinning of dislocations and recovery at elevated temperatures, and on the interaction of dislocations with grown-in subboundaries.

INTRODUCTION

THE indentation hardness test is probably the simplest method of measuring the strength of materials. However, it is also the least understood test in terms of stress and strain distribution. Some progress has been made in analyzing the stress and strain distributions of several types of indentations using the continuum theory of plasticity.[1] However, the results cannot be applied directly to crystalline solids having well-defined slip systems.

In the last decade, a few attempts have been made to study the deformation mechanism associated with indentation. Tolansky and Nickols[2] studied several matrials by means of multiple-beam interference microscopy. Churchman, Geach, and Winston[3] investigated materials with a diamond structure. Smakula and Klein[4] used a prismatic punching method to study glide in ionic crystals. Votava, Amelinckx, and Dekeyser[5] employed an interferometric method to study indentation figures on cleavage faces of mica and NaCl.

With the advancement of dislocation theory and techniques of revealing dislocations in crystals in the past few years, it was thought possible to attain a better understanding of the deformation caused by indentation of a material with a simple crystalline structure. In this investigation, dislocation etching technique[6,7] was used to study the dislocation structures associated with various types of indentations at various temperatures in magnesium oxide crystals. This type of study may

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